modes increases. On the other hand, the contribution of the first mode to the total response is small when λ is small and increases with λ. This indicates that as λ increases the contribution of the lower modes increases, i.e., C_{mn} decreases with λ by a smaller amount for m and n small than for m and n large. As θ increases, the power spectral density of the forcing pressure shifts to the lower frequencies. Thus, the contribution of the first mode to the total response increases with an increase in θ. One also observes that G_a increases with θ, indicating that the modes become uncorrelated with an increase in θ.

Knowing the correlation of the pressure field, now one has a basis for neglecting the cross correlation between the modes or considering the first mode only in calculating the response of a structure.

The foregoing conclusions are valid only for the correlation functions that were assigned in Eqs. (10–12). These correlation functions encompass a wide range of physical cases. The discussion does not apply for convective pressure fields. In the latter situations, coincidence might play a significant role and change the picture.

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Time-Dilatation Dilemma and Scale Variation

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1. Transformation Equations

We assume the special theory of relativity in the sense of Einstein. Let us assume two inertial coordinate systems, S and S^*. S^* moves relatively to S at the constant rate v along the X axis. At the S time $t = 0$, the points of origin of S and S^* coincide. The X^* axis coincides with the X axis. The transformation equations are

$$x^* = (x - vt)(1 - v^2/c^2)^{-1/2} \qquad y^* = y \qquad z^* = z \tag{1.1}$$

$$t^* = (t - vc^{-2}x)(1 - v^2/c^2)^{-1/2} \tag{1.2}$$

The inverse transformation equations are

$$x = (x^* + vt^*)(1 - v^2/c^2)^{-1/2} \qquad y = y^* \qquad z = z^* \tag{1.3}$$

$$t = (t^* + vc^{-2}x^*)(1 - v^2/c^2)^{-1/2} \tag{1.4}$$

2. Usual Formulation of Time-Dilatation Dilemma

At the present time there exist various forms of the interpretation of the time dilatation (the same refers to the length contraction). Let us consider a clock that is rigidly connected with the starred frame of reference and stationed at some point (x_0^*, y_0^*, z_0^*). Let us compare the time indicated by that clock with the time t measured in the unstarred system. An S time interval, $(t_2 - t_1)$, is therefore related to the readings t_2^* and t_1^* as follows:

$$t_2 - t_1 = (t_2^* - t_1^*)(1 - v^2/c^2)^{-1/2} \tag{2.1}$$

In a similar way we obtain

$$t_2^* - t_1^* = (t_2 - t_1)(1 - v^2/c^2)^{-1/2} \tag{2.2}$$

and the Lorentz contractions referring to lengths. The rules governing these phenomena are given by Bergmann[1] in the following forms:

> Every clock appears to go at its fastest rate when it is at rest relatively to the observer. If it moves relatively to the observer with the velocity v, its rate appears slowed down by the factor $(1 - v^2/c^2)^{-1/2}$. Every rigid body appears to be the longest when at rest relatively to the observer. When it is not at rest, it appears to be contracted in the direction of its relative motion by the factor $(1 - v^2/c^2)^{-1/2}$, while its dimensions perpendicular to the direction of motion are unaffected.[1]

Tolman[14] concludes that the time interval between two events (which occur at the same point in S^*), which has the duration dt^* when measured with a given clock in system S^*, will have the longer duration of $dt^*\ (1 - v^2/c^2)^{-1/2}$ when measured by the clocks in system S. Similarly, the time interval between two events occurring at the same point in S, which has duration dt when measured with a given clock in the system S, has the longer duration of $dt(1 - v^2/c^2)^{-1/2}$ when measured with the clocks in the system S^*. One may refer to the recent controversy on the subject of time dilatation and to the works and notes of Born,[2] Dingle,[3–6] Palacios,[9–11] Pilgeram,[12] and others. From the more interesting suggestions one should mention the proposition of Schlegel[13] concerning a distinction between the macroscopic thermodynamic processes, called Clasius processes, which are time invariant and independent of the relativistic transformations, and the Lorentz processes, which are subject to the relativistic transformations. Pilgeram[12] introduces the concept of the biological time, independent of the consequences of Einstein's theory of relativity, in his discussion of the Hoerner paper[7] in which Hoerner concluded that 60 years for a crew member aboard a rocket attempting an interstellar trip will be equivalent to a life span of 5×10^6 years on earth.

It is obvious that any statement in the sense that a uniform motion of a clock will exert some influence upon the internal mechanism of the clock, so that it will slow down or accelerate, is a misleading one. The phenomenon of slowing down or accelerating a clock is a dynamic phenomenon that must involve the action of some sort of forces. The special theory of relativity can obviously refer only to the "kinematic" effects with no action of forces caused by the relative motion. The same refers to the length contraction.

3. Scale Variation in the Time-Dilatation Dilemma

We accept the validity of the Schlegel hypothesis; consequently, the discussion given below refers to phenomena that depend upon the space-time relativistic transformations. Moreover, we assume that, in order to preserve the validity of the fundamental concepts of the special theory of relativity, we allow for the change of the scale (i.e., of a unit) in the systems in question. This is nothing new, since, in various other fields of mathematical physics, the problem of the variation of scale of coordinates is a well known and popular one.† Assume two clocks, one in system S and another one in S^*. We would like to synchronize these two clocks so

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† In gasdynamics there are widely used the so-called similarity rules (by Prandtl-Glauert, Kármán-Tsien and others), where there appears a factor $(1 - v^2/a^2)^{-1/2}$, a = velocity of sound, similar to the relativistic factor. To comply with the physical reality, the scale in one of the two systems in question must be appropriately changed.

that one of them would indicate to us the time on another clock at a given moment. We assume that the only possible communication correlation (correspondence) between these two clocks located in S and S^* is by means of light signals (Lorentz systems). Assume the clock in S^* system located at $x_0^* = 0$. From Eq. (1.1) it is seen that this corresponds to the magnitude $x = vt$ in S system. Inserting this value of x into Eq. (1.2) gives the result

$$t^* = t(1 - v^2/c^2)^{1/2}$$
$$t_2 - t_1 = (t_2^* - t_1^*)(1 - v^2/c^2)^{-1/2} \tag{3.1}$$

For $v^2 = 0$, one has $t_2^* - t_1^* = t_2 - t_1$, which is correct. In order to synchronize both clocks we reason in the following way: Assume that the time interval $(t_2 - t_1)$ on the clock in S system is equal to a chosen unit of time (second, minute, hour). In order that the clock located in S^* be synchronized with the clock in S, i.e., in order that the clock in S^* would show us precisely what time there is on the clock C in S at a given moment from the relativistic standpoint, the unit time-interval $(t_2^* - t_1^*)$ of the clock in S^* must be equal to $(1 - v^2/c^2)^{1/2}$, i.e., must be smaller than $(t_2 - t_1)$, which is equal to a unit. It is assumed that the internal mechanisms of clocks do not belong to a Lorentz family of systems; they belong to a Clausius family of systems in Schlegel's sense. This means that the light signals do not effect the action or the rate of action of the mechanisms of clocks in both S and S^*. Then this implies that for our space trips with very large velocities we have to construct clocks having scales different from scales of the clocks used ordinarily on the earth, i.e., for a synchronization of both clocks in S and S^* we have to have

$$\text{unit of time of a clock in the spaceship} = (1 - v^2/c^2)^{1/2} \tag{3.2}$$

We conclude: Under the assumption that the fundamental principles of the classical special theory of relativity are valid, if a clock C^*, whose internal mechanism belongs to a Clausius family of systems, moves relatively to a clock C located on the earth (C belongs to a Clausius system) with a velocity v, and which is in some sort of a synchronization with C by means of only light signals, which do not affect the action or the rate of action of both clocks, C and C^*, then the (relativistic) unit of time of the clock C^* must be equal to the factor $(1 - v^2/c^2)^{1/2}$ to be in a permanent synchronization with the clock C. The internal mechanism of the clock C^* must be correspondingly designed and constructed to indicate the time synchronized with the time of the clock C so that the time scale of the clock C^* be different from the time scale of the clock C (on the earth).

The foregoing reasoning can be illustrated briefly by the following example: Suppose the spaceship leavest he earth at 3:00 p.m. and both clocks, C and C^*, indicate precisely 3:00 p.m. Suppose that the factor $(1 - v^2/c^2)^{1/2}$ is equal to $\frac{1}{2}$. The unit time of the clock C^* is equal to one-half of the unit time of the clock C, i.e., the clock C^* runs faster than the clock C. Suppose that at 4:00 p.m. on the clock C^* we send a signal to C that it is 4:00 p.m. Actually, at this moment (from the absolute point of view) it is only 3:30 p.m. on the clock C. But the light signal from C^* will reach the observer on the earth and the clock C precisely at 4:00 p.m. In this example the receiving clock C has the same scale as all the other clocks in the S system (on the earth), and the signal-sending clock C^* has the scale changed.

Let us now discuss the inverse case. Assume a clock D located in S system of coordinates at $x = 0$. From Eq. (1.3) we have the corresponding magnitude of x^* equal to $x^* = -vt^*$, which, when inserted into Eq. (1.4), furnishes

$$t = t^*(1 - v^2/c^2)^{1/2}$$
$$(t_2^* - t_1^*) = (t_2 - t_1)(1 - v^2/c^2)^{-1/2} \tag{3.3}$$

We apply the following reasoning: Assume that the time interval $(t_2^* - t_1^*)$ on the clock D^* in S^* system is equal to a unit of time on D^* (second, minute, hour). In order that the clock located in S be synchronized with the clock in S^*, i.e., in order that the clock D in S would show us precisely what time there is on the clock D^* in S^* at a given moment from the relativistic standpoint, the unit time-interval $(t_2 - t_1)$ of the clock D must be equal to $(1 - v^2/c^2)^{1/2}$, i.e., must be smaller than $(t_2^* - t_1^*)$, which is equal to a unit. As in the foregoing, it is assumed that the clock interval mechanisms belong to a Clausius family of systems. We conclude: Under the assumption that the fundamental principles of the classical special theory of relativity are valid, if a clock D^*, whose internal mechanism belongs to a Clausius family of systems, moves relatively to a clock D located on the earth with a velocity v and which is in some sort of a synchronization with D by means of only light signals, which do not affect the action or the rate of action of both clocks D and D^*, then the (relativistic) unit of time of the clock D must be equal to the factor $(1 - v^2/c^2)^{1/2}$ to be in a permanent synchronization with the clock D^*.

The following example may be quoted. Suppose, the spaceship leaves the earth at 3:00 p.m., and both clocks D and D^* indicate precisely 3:00 p.m. With the factor $(1 - v^2/c^2)^{1/2}$ equal to $\frac{1}{2}$, the unit of time of D is equal to one-half of the unit time of all the other clocks on the earth and of the clock D^*, i.e., the clock D runs faster than all the other clocks on the earth. Suppose that at 4:00 p.m. on D we send a signal to D^* that it is 4:00 p.m. Actually, at this moment it is only 3:30 p.m. on all of the other clocks on the earth and on the clock D^*. But the light signal from D will reach the observer in the spaceship and the clock D^* precisely at 4:00 p.m. In this example the signal-sending clock D has the scale changed, whereas the receiving clock D^* has the same scale as all the other clocks on the earth. Hence, a distinction must be made between the signal-sending clocks C^* and D, and the signal-receiving clocks, C and D^*.

The foregoing proposition of the various time scales of the signal-sending and signal-receiving clocks seems to explain the so-called clock paradox. The dilemma of the length contraction is treated in another paper by the author.

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